FORM TP 2019090

TEST CODE **01234020**

MAY/JUNE 2019

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of TWO sections: I and II.
2. Section I has SEVEN questions and Section II has THREE questions.
3. Answer ALL questions.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. All working MUST be clearly shown.
7. **A list of formulae is provided on page 4 of this booklet.**
8. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
9. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Required Examination Materials

Electronic calculator
Geometry set

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION I

Answer ALL questions.

All working must be clearly shown.

1. (a) Using a calculator or otherwise, evaluate EACH of the following:

(i) $\dfrac{2\frac{1}{4} - 1\frac{3}{5}}{3}$

$\dfrac{5 - 12}{20} = \dfrac{13}{20}$

$\dfrac{13}{20} \div \dfrac{3}{1}$

$\dfrac{13}{20} \times \dfrac{1}{3} = \dfrac{13}{60}$

ANS: $\dfrac{13}{60}$

(2 marks)

(ii) 2.14 sin 75°, giving your answer to 2 decimal places

$2.14 \times 0.966 = 2.07$

2.07 (2 d.p.)

(1 mark)

(b) Irma's take-home pay is $4 320 per fortnight (every two weeks). Each fortnight Irma's pay is allocated according to the following table.

Item	Amount Allocated
Rent	$\$x$
Food	\$629
Other living expenses	$\$2x$
Savings	\$1 750
Total	**\$4 320**

(i) What is Irma's **annual** take-home pay? (Assume she works 52 weeks in any given year.)

No. of fortnights: $\frac{52}{2} = 26$ fortnights.

1 fortnight = \$4320.00

26 fortnights: \$4320 × 26 = \$112320.00

\$112320.00.

(1 mark)

(ii) Determine the amount of money that Irma allocates for rent each month.

$\$x + \$2x + \$629 + \$1750 = \$4320.$

$\$3x + \$2379 = \$4320.$

$3x = \$4320 - \$2379 = \$1941.$

$x = \frac{\$1941.}{3} = \$647.00$

RENT = $647.00

(3 marks)

(iii) All of Irma's savings is used to pay her son's university tuition cost, which is $150 000.

If Irma's pay remains the same and she saves the same amount each month, what is the MINIMUM number of years that she must work in order to save enough money to cover her son's tuition cost?

Savings $1750.

Savings for the year: $\$1750 \times 26 = \$45,500.00$

No. of years to cover her son's tuition:

$\frac{\$150,000}{\$45,5000}$

$= 3.29$ years

∴ Minimum number of years = 4 years.

ANS: 4 years.

(2 marks)

Total 9 marks

2. (a) Simplify completely

(i) $3p^2 \times 4p^5$

$12p^{2+5} = \underline{\underline{12p^7}}$

$12p^7$

(1 mark)

(ii) $\frac{3x}{4y^3} \div \frac{21x^2}{20y^2}$.

$\frac{\cancel{3x}}{\cancel{4}yyy} \times \frac{\cancel{20}yy}{\cancel{21}x\,x} = \frac{5}{7xy}$

ANS = $\frac{5}{7xy}$

(2 marks)

(b) Solve the equation

$$\frac{3}{7x-1} + \frac{1}{x} = 0.$$

$$\frac{3x + 1(7x-1)}{(7x-1)x} = \frac{0}{1}$$

$$3x + 7x - 1 = 0.$$

$$10x - 1 = 0$$

$$10x = 1$$

$$x = \frac{1}{10}$$

ANS = $\frac{1}{10}$

(3 marks)

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(c) When a number, x, is multiplied by 2, the result is squared to give a new number, y.

(i) Express y in terms of x.

$(2x)^2 = y$

$y = 4x^2$

ANS: $y = 4x^2$.

(1 mark)

(ii) Determine the two values of x that satisfy the equation $y = x$ AND the equation derived in (c) (i).

If $y = x$

$4x^2 = x$

then $4x^2 - x = 0.$

$x(4x-1) = 0.$

$x = 0 \quad 4x - 1 = 0$

$4x = 1$

$x = \frac{1}{4}.$

Equation: $4x^2 - x = 0.$

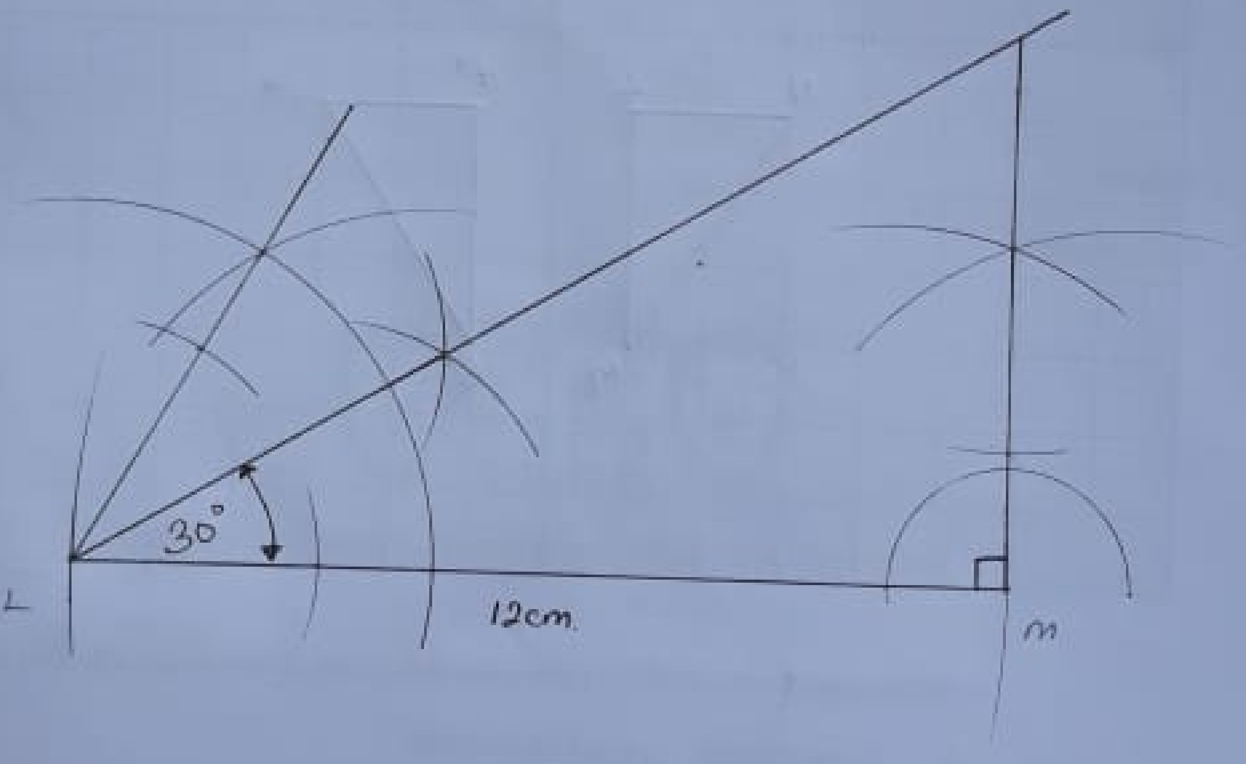
$x = 0, \ x = \frac{1}{4},$ $\boxed{\text{Eqn: } 4x^2 - x = 0}$

(2 marks)

Total 9 marks

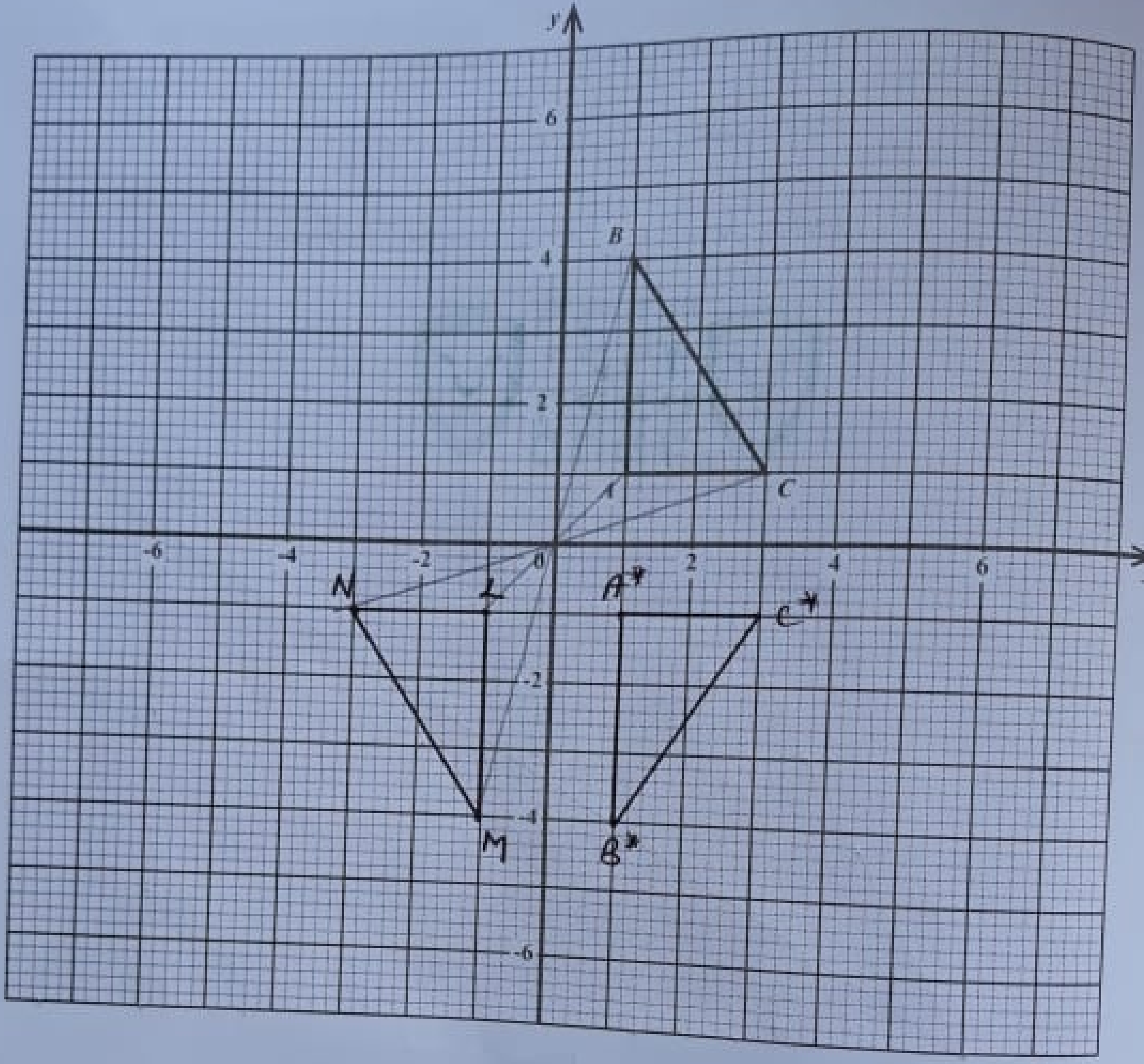
3. (a) Using a ruler, a pencil and a pair of compasses only, construct the triangle NLM, in which $LM = 12$ cm, $\angle MLN = 30°$ and $\angle LMN = 90°$.

(Credit will be given for clearly drawn construction lines.)



(4 marks)

(b) Triangle ABC with vertices A (1, 1), B (1, 4) and C (3, 1) is shown on the diagram below.



ΔABC is mapped onto ΔLMN by a reflection in the x-axis followed by a reflection in the y-axis.

(i) On the diagram, draw and label ΔLMN.

(2 marks)

(ii) Describe fully a single transformation that maps ΔABC onto ΔLMN.

A counter-clockwise or clockwise rotation of 090° about the Origin (0, 0).

(2 marks)

(iii) State the 2×2 matrix for the transformation that maps ΔABC onto ΔLMN.

Rotation ~~of~~ Centre (0, 0) through 180°:

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$R_{180°} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(1 mark)

Total 9 marks

4. (a) The quantity P varies inversely as the square of V.

(i) Using the letters P, V and k, form an **equation** connecting the quantities P and V.

$P \propto \frac{1}{V^2}$ $k = 4 \times 3^2 = 4 \times 9 = 36.$

$P = \frac{k}{V^2}$ $\therefore P = \frac{\quad}{V^2}$

$\frac{4}{1} = \frac{k}{3^2}$

$P = \frac{36}{V^2}$

(1 mark)

(ii) Given that $V = 3$ when $P = 4$, determine the positive value of V when $P = 1$.

$P = \frac{36}{V^2}$

$\frac{1}{1} = \frac{36}{V^2}$

$V^2 = 36$

$V = \pm\sqrt{36} = \pm 6.$

$V = \pm 6. \therefore$ Positive Value $= +6.$

(2 marks)

(b) (i) Given that x is a real number, solve the inequality

$$-7 < 3x + 5 \leq 7.$$

R.H.S. $3x + 5 \leq 7.$

$3x \leq 7 - 5$

$3x \leq 2$

$x \leq \frac{2}{3}.$

L.H.S. $-7 < 3x + 5$

$-7 - 5 < 3x$

$-12 < 3x$

$\frac{-12}{3} < x$

$-4 < x$

$$\boxed{-4 < x \leq \frac{2}{3}.}$$

ANS: $-4 < x \leq \frac{2}{3}.$

(2 marks)

(ii) Represent your answer in (b) (i) on the number line shown below.

(1 mark)

(c) The equation of a straight line is given as

$$\frac{x}{3} + \frac{y}{7} = 1.$$

This line crosses the y-axis at Q.

(i) Determine the coordinates of Q.

If line crosses the y axis, then $x = 0$.

$\frac{0}{3} + \frac{y}{7} = \frac{1}{1}$

$\frac{y}{7} = \frac{1}{1}$, $y = 7$

Co-ordinates at $Q = (0, 7)$.

$Q = (0,7)$

(1 mark)

(ii) What is the gradient of this line?

$\frac{x}{3} + \frac{y}{7} = \frac{1}{1}$

$\frac{7x + 3y}{21} = \frac{1}{1}$

$7x + 3y = 21.$

$3y = -7x + 21$

$y = \frac{-7x + 21}{3} = \frac{-7}{3}x + 7.$

$m = \frac{-7}{3}.$

(2 marks)

Total 9 marks

5. The cumulative frequency distribution of the volume of petrol needed to fill the tanks of 150 different vehicles is shown below.

Volume (litres)	Cumulative Frequency
11–20	24
21–30	59
31–40	101
41–50	129
51–60	150

(a) For the class 21–30, determine the

(i) lower class boundary

The lower class boundary of the 1st class Interval:

$$\frac{\text{The Upper class limit of the first class} + \text{The lower class limit of the second class}}{2}$$

$$= \frac{20+21}{2} = \underline{\underline{20.5}}$$

ANS: 20.5.

(1 mark)

(ii) class width.

The Width of a class Interval: The Upper Class Boundary − The lower class Boundary.

$$= \left(\frac{40+41}{2}\right) - \frac{30+31}{2}$$

$$\Rightarrow 40.5 - 30.5 = 10.$$

Class Width: 10.

(1 mark)

(b) How many vehicles were recorded in the class 31–40?

10 Vehicles.

10.

(1 mark)

(c) A vehicle is chosen at random from the 150 vehicles. What is the probability that the volume of petrol needed to fill its tank is **more** than 50.5 litres? **Leave your answer as a fraction.**

Volume	Frequency	Cumulative Frequency
10.5 – 20.5	24	24
20.5 – 30.5	35	59
30.5 – 40.5	42	101
40.5 – 50.5	28	129.
50.5 – 60.5	21	150.

$\frac{21}{150} = \frac{7}{50}.$

(2 marks)

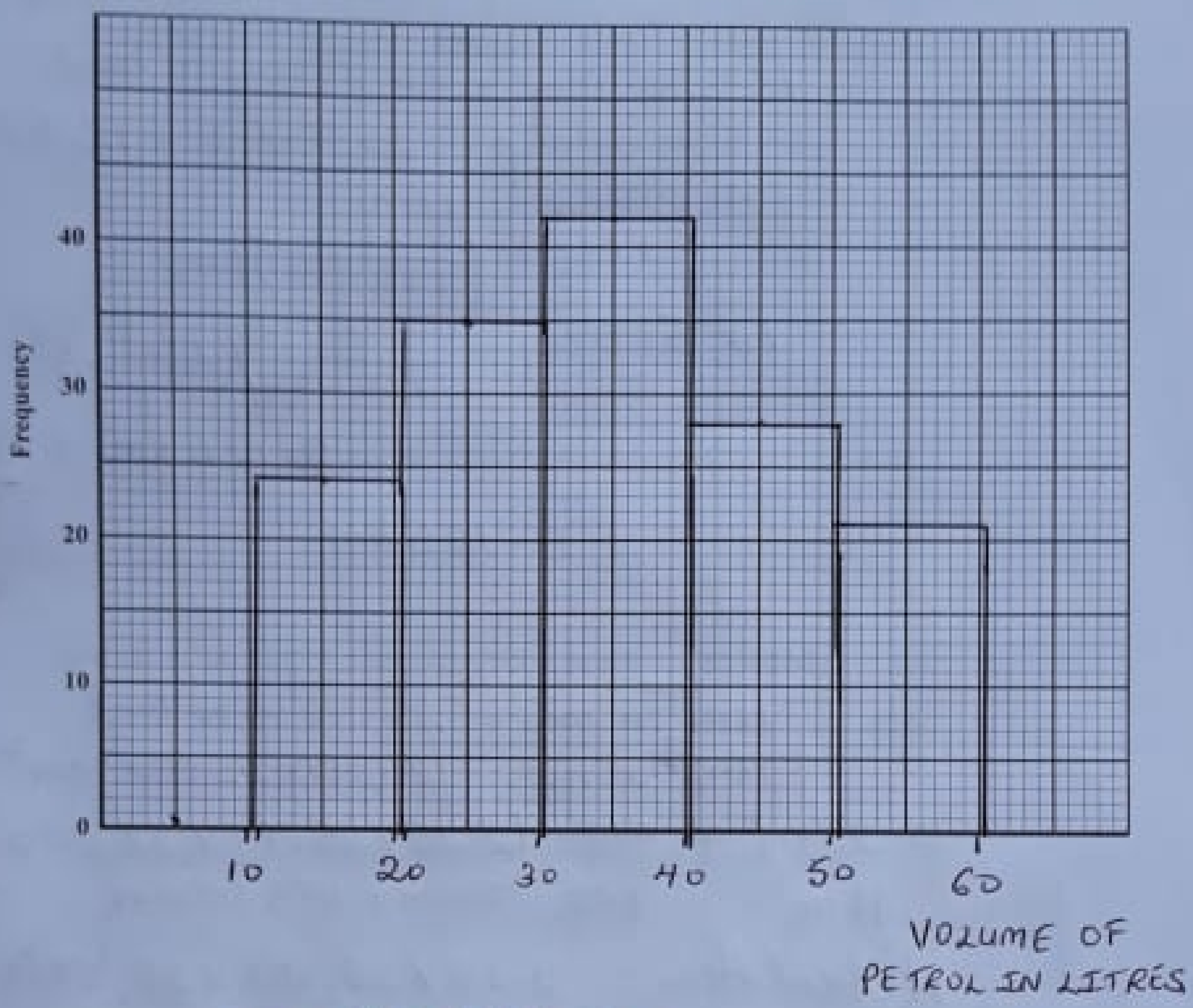
(d) Byron estimates the median amount of petrol to be 43.5 litres. Explain why Byron's estimate is INCORRECT.

The Median on the Cumulative Frequency : $\frac{1}{2}(150)$ $= 75$

According to the table $75 < 10\cancel{1}$ whose volume corresponds to 31–40L, Hence an answer of 43.5L would be too high and hence incorrect.

(1 mark)

(e) On the partially labelled grid below, construct a histogram to represent the distribution of the volume of petrol needed to fill the tanks of the 150 vehicles.



(3 marks)

Total 9 marks

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6. (a) The scale on a map is 1:25 000.

(i) Determine the actual distance, in km, represented by 0.5 cm on the map.

1 cm on map = 25,000 cm on land.
0.5 cm on map = 25,000 X .5 = 12,500 cm
100,000 cm = 1 km
1 cm = $\left(\frac{1}{100,000}\right)$ km
12,500 cm = $\frac{1}{100,0\cancel{00}} \times \frac{12,5\cancel{00}}{1}$
= 0.125 km or $\frac{1}{8}$ km.

0.125 km.

(2 marks)

(ii) Calculate the actual area, in km^2, represented by 2.25 cm^2 on the map.

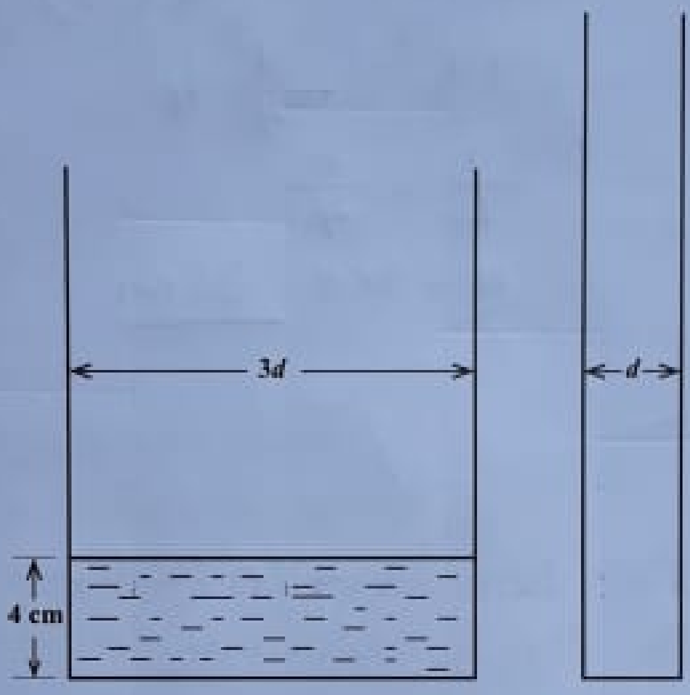
1 cm on map = 25,000 cm on land.
1 cm^2 on map = $(25,000 \times 25000)\ cm^2$.
2.25 cm^2 on map = $(2.25 \times 25000 \times 25,000)\ cm^2$.
1 km^2 = 100,000 cm x 100,000 cm.
Actual Area: $\frac{2.25 \times 25,\cancel{000} \times 25,\cancel{000}}{100,\cancel{000} \times 1\cancel{00},\cancel{000}}$
= $\frac{1406.25}{10,000}$ = 0.140625 km^2

Actual Area: 0.140625 km^2.

(3 marks)

(b) The diagram below **(not drawn to scale)** shows the cross-section of two cylindrical jars, Jar X and Jar Y. The diameters of Jar X and Jar Y are $3d$ cm and d cm respectively.

Initially, Jar Y is empty and Jar X contains water to a height (depth) of 4 cm.



(i) Determine, in terms of π and d, the volume of water in Jar X.

$r = \frac{3d}{2}$ cm, $V = \pi \left(\frac{3d}{2}\right)^2 \times 4$ $[V = \pi r^2 h]$

$V = \pi \times \frac{9d^2}{\cancel{4}} \times \frac{\cancel{4}}{1} = 9d^2\pi \text{ cm}^3$

$V = 9d^2\pi \text{ cm}^3.$

(2 marks)

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(ii) If all the water from Jar X is now poured into Jar Y, calculate the height it will reach.

Vol. of Jar X = Vol. of Jar Y.

$$= \quad 9d^2\cancel{\pi} = \cancel{\pi}\left(\frac{d}{2}\right)^2 h.$$

$$9\cancel{d}^2 = \frac{\cancel{d}^2}{4}h$$

$$\frac{9}{1} = \frac{h}{4}$$

$$h = 9 \times 4 = \underline{36 \text{ cm}}$$

height of JAR Y = 36 cm.

(2 marks)

Total 9 marks

7. (a) The nth term, T_n, of a sequence is given by

$$T_n = 3n^2 - 2.$$

(i) Show that the first term of the sequence is 1.

$T_n = 3n^2 - 2, \; n = 1.$
$3(1)^2 - 2 = 3 - 2 = 1.$ Q.E.D.

(1 mark)

(ii) What is the third term of the sequence?

$T_n = 3n^2 - 2, \; n = 3$
$3(3)^2 - 2 = 3(9) - 2 = 27 - 2 = \underline{\underline{25}}$

(1 mark)

(iii) Given that $T_n = 145$, what is the value of n?

$3n^2 - 2 = 145$
$3n^2 = 145 + 2 = 147.$
$n^2 = \frac{147}{3} = 49$
$n = \sqrt{49} = \underline{7}$

$n = 7.$

(3 marks)

(b) The first 8 terms of another sequence with n^{th} term, $U(n)$, are

*Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, 21

where

$U(1) = 1$
$U(2) = 1$ and
$U(n) = U(n-1) + U(n-2)$ for $n \geq 3$.

For example, the fifth and seventh terms are

$U(5) = U(4) + U(3) = 3 + 2 = 5$
$U(7) = U(6) + U(5) = 8 + 5 = 13$.

(i) Write down the next two terms in the sequence, that is, $U(9)$ and $U(10)$.

Using the Fibonacci Mechanism:
$u_9 = u_8 + u(7)$
$= 21 + 13 = 34$.
$u_{10} = u_9 + u_8 = 34 + 21 = 55$

(2 marks)

(ii) Which term in the sequence is the sum of $U(18)$ and $U(19)$?

$\Sigma u_{18} = 6764$. $\Sigma u_{19} = 10,945$.

Using the Sequence

= +

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610,
16 17 18 19
987, 1597, 2584, 4181

$u(18) = 6764$ & $u(19) = 10,945$.

(1 mark)

(iii) Show that $U(20) - U(19) = U(19) - U(17)$.

R.T.P.

$u(20) - u(19) = u(19) - u(17)$

$u(20) = u(19) + u(18)$

$= 6765$.

Now

$6765 - 4181 = 4181 - 1597$.

$2584 = 2584$.

Q.E.D.

(2 marks)

Total 10 marks

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SECTION II

Answer ALL questions.

All working must be clearly shown.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. (a) The functions f and g are defined by

$$f(x) = \frac{9}{2x+1} \quad \text{and} \quad g(x) = x - 3.$$

(i) State a value of x that CANNOT be in the domain of f.

In other words, the function can be UNDEFINED

$\therefore\ 2x+1 = 0$

$2x = -1$

$x = \frac{-1}{2}$

$x = -\frac{1}{2}.$

(1 mark)

(ii) Find, in its simplest form, expressions for

a) $fg(x)$

$f(x) = \frac{9}{2x+1}.$

$f(g) = \frac{9}{2g+1} = \frac{9}{1} \div 2(x-3)+1$

$\frac{9}{1} \div \frac{2x-6+1}{1}$

$\frac{9}{1} \times \frac{1}{2x-5} = \frac{9}{2x-5}$

$fg(x) = \frac{9}{2x-5}.$

(2 marks)

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b) $f^{-1}(x)$.

$$f(x) = \frac{9}{2x+1}$$

$$y = \frac{9}{2x+1}$$

$$\frac{x}{1} = \frac{9}{2y+1}$$

$$x(2y+1) = 9$$

$$2y+1 = \frac{9}{x}$$

$$2y = \frac{9}{x} - 1.$$

$$y = \frac{1}{2}\left(\frac{9}{x} - 1\right) = \frac{9}{2x} - \frac{1}{2}$$

OR $2xy + x = 9$

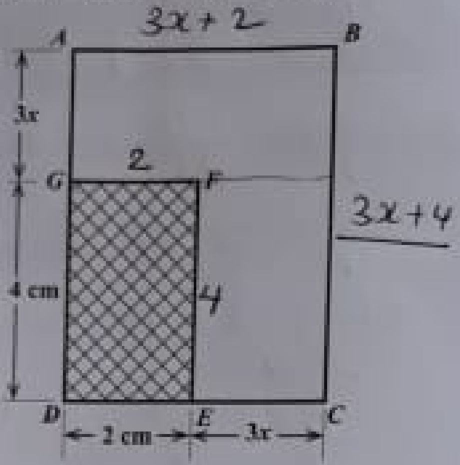
$$2xy = 9 - x$$

$$y = \frac{9-x}{2x} = f^{-1}(x) = \frac{9-x}{2x}$$

$$f^{-1}(x) = \frac{9-x}{2x}.$$

(2 marks)

(b) The diagram below shows two rectangles, $ABCD$ and $GFED$. $ABCD$ has an area of 44 cm^2. $GFED$ has sides 4 cm and 2 cm. $AG = EC = 3x$ cm.



(i) By writing an expression for the area of rectangle $ABCD$, show that $x^2 + 2x - 4 = 0$.

$AB \cdot DC = 44 \text{ cm}^2$

$(3x+4)(3x+2) = 44$

$9x^2 + 6x + 12x + 8 = 44$

$9x^2 + 18x + 8 - 44 = 0.$

$\frac{1}{9}(9x^2 + 18x - 36 = 0)$

$= x^2 + 2x - 4 = 0$

Q.E.D

(3 marks)

(ii) Calculate, to 3 decimal places, the value of x.

$a = 1,\ b = 2,\ c = -4$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-2 \pm \sqrt{4+16}}{2}$

$\dfrac{-2 \pm 2\sqrt{5}}{2} = \dfrac{\cancel{2}(-1 \pm \sqrt{5})}{\cancel{2}}$

$x = \dfrac{-1 + 2.236}{}$, $x = -1 - 2.236$

$x = 1.236$, $x = -3.236.$

$\boxed{x = 1.236,\ x = -3.236.}$

(2 marks)

(iii) Calculate the perimeter of the UNSHADED region.

Total Perimeter:

$\underline{3x} + 2 + \underline{3x} + 4 + \underline{3x} + 4 + 2 + \underline{3x} = 12x + 12$

$x = +1.236$ cm

then $12(1.236) + 12 = \underline{\underline{26.832\text{cm}}}.$

ANS: 26.832cm.

(2 marks)

Total 12 marks

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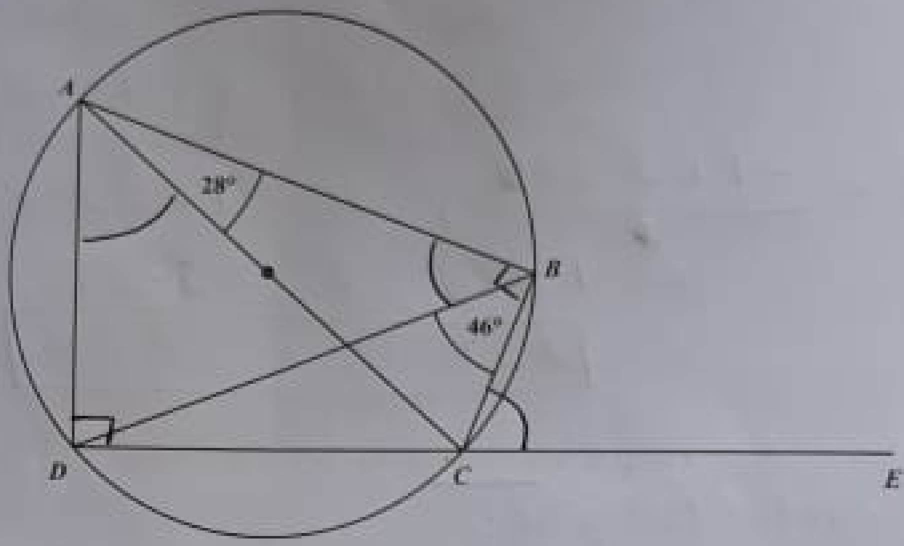
GEOMETRY AND TRIGONOMETRY

9. (a) The diagram below shows a circle where AC is a diameter. B and D are two other points on the circle and DCE is a straight line. Angle $CAB = 28°$ and $\angle DBC = 46°$.



Calculate the value of each of the following angles. Show detailed working where necessary and give a reason to support your answers.

(i) $\angle DBA$

Since DC is a Diameter, the diameter, DC, subtends an angle of 90°.

$\therefore$ DBA = 90° − 46° = 44°

Reason Diameter DC subtends an angle of 90°.

(2 marks)

(ii) $\angle DAC$ $= \underline{46^\circ}$

Reason Angles $D\hat{A}C$, and $D\hat{B}C$ are the same, since they are both subtended by the same chord, DC.

(2 marks)

(iii) $\angle BCE$ $= \underline{74}^\circ$.

— ABCD is a Cyclic Quadrilateral. Opposite angles add up to $\underline{180^\circ}$

$\therefore \angle BCD = 180^\circ - (28 + 46^\circ) = \underline{106^\circ}$

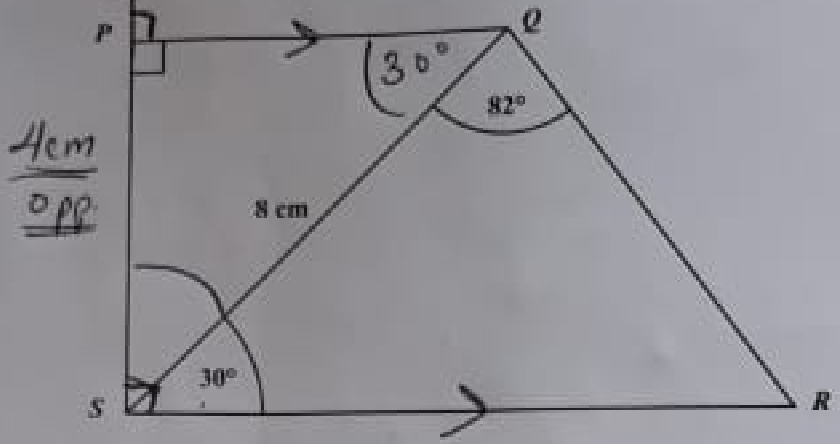
— Since DCE is a straight line ($\Sigma \angle's = 180^\circ$)

$\therefore B\hat{C}E = 180^\circ - 106^\circ = \underline{74^\circ}$

Reason Opposite Angles in a Cyclic Quadrilateral $= 180^\circ$ / DCE is a ~~st~~ straight line (Sum of angles $= 180^\circ$).

(2 marks)

(b) The diagram below shows a quadrilateral $PQRS$ where PQ and SR are parallel, $SQ = 8$ cm, $\angle SPQ = 90°$, $\angle SQR = 82°$ and $\angle QSR = 30°$.



Determine

(i) the length PS

$\angle PQS = 30°$ (Alt. Angles since $PQ \parallel SR$.)

$\frac{PS}{8} = \frac{\sin 30°}{1}$

$PS = 8 \sin 30° = 8\left(\frac{1}{2}\right) = \underline{4\text{ cm}}$.

$PS = 4$ cm.

(2 marks)

(ii) the length PQ

$$\frac{PQ}{8} = \frac{\cos 30^\circ}{1}$$

$$\frac{PQ}{8} = \frac{\sqrt{3}}{2}$$

$$2PQ = 8\sqrt{3}$$

$$PQ = \frac{8\sqrt{3}}{2} = 4\sqrt{3} = 4 \times 1.73 = \underline{6.92\text{cm}}.$$

$PQ = 6.92\text{cm}.$

(1 mark)

(iii) the area of $PQRS$. In ΔQSR, $\angle R = 180^\circ - (82^\circ + 30^\circ)$

$180^\circ - 112^\circ = \underline{68^\circ}$

Using Sine Rule : $\dfrac{SR}{\sin 82^\circ} = \dfrac{8\text{cm}}{\sin 68^\circ}$

$= SR \sin 68^\circ = 8 \times \sin 82^\circ$

$SR = \dfrac{8 \times \sin 82^\circ}{\sin 68} =$

$SR = \dfrac{8 \times 0.99}{0.927} = \underline{8.5\text{cm}}.$

Area of trapezium $PQRS = \frac{1}{2} \Sigma \| s \perp h.$

$= \frac{1}{2}(6.92 + 8.5)4$

$2(15.42) = 30.84\text{cm}^2$

$A = 30.8\text{cm}^2.$

(3 marks)

Total 12 marks

VECTORS AND MATRICES

10. (a) (i) a) Find the matrix product

$$\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix} \begin{pmatrix} k \\ 5 \end{pmatrix}.$$

$$\begin{pmatrix} -k + 15 \\ 4k + 5h \end{pmatrix}$$

(2 marks)

b) Hence, find the values of h and k that satisfy the matrix equation

$$\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix} \begin{pmatrix} k \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

$-k + 15 = 0.$

$+k = +15$

$k = 15$

$4k + 5h = 0.$

$4(15) + 5h = 0.$

$60 + 5h = 0$

$5h = -60$

$h = \frac{-60}{5} = \underline{\underline{-12}}$

$$\boxed{\begin{matrix} k = 15 \\ h = -12 \end{matrix}}$$

$k = 15, \ h = -12.$

(2 marks)

(ii) Using a matrix method, solve the simultaneous equations

$$2x + 3y = 5$$
$$-5x + y = 13.$$

$$\begin{pmatrix} 2 & 3 \\ -5 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 13 \end{pmatrix}$$

$$2+15 = 17 \quad = \frac{1}{17}\begin{pmatrix} 1 & -3 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{17} & -\frac{3}{17} \\ \frac{5}{17} & \frac{2}{17} \end{pmatrix}$$

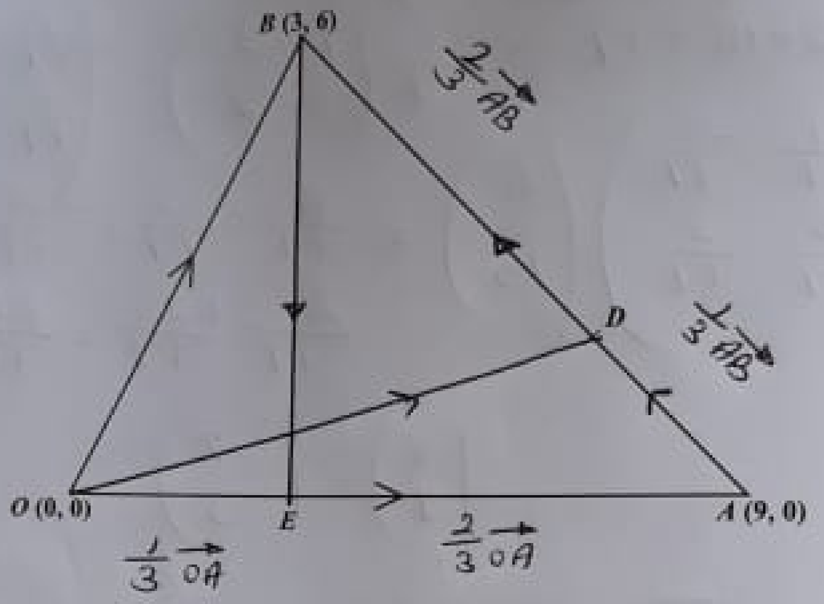
$$\begin{pmatrix} \frac{1}{17} & \frac{-3}{17} \\ \frac{5}{17} & \frac{2}{17} \end{pmatrix}\begin{pmatrix} 5 \\ 13 \end{pmatrix} = \begin{matrix} \frac{5}{17} - \frac{39}{17} = \frac{-34}{17} = -2 \\ \frac{25}{17} + \frac{26}{17} = \frac{51}{17} = 3. \end{matrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}.$$

$\boxed{x = -2,\ y = 3}$

(3 marks)

(b) Relative to the origin $O(0, 0)$, the position vectors of the points A and B are $OA = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$ and $OB = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$ respectively. The points D and E are on AB and OA respectively and are such that $AD = \frac{1}{3} AB$ and $OE = \frac{1}{3} OA$. The following diagram illustrates this information.



Express the following vectors in the form $\begin{bmatrix} a \\ b \end{bmatrix}$.

(i) $\overrightarrow{AB}$

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$
$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= \begin{pmatrix} 3 \\ 6 \end{pmatrix} - \begin{pmatrix} 9 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 6 \end{pmatrix}.$$

$$\overrightarrow{AB} = \begin{pmatrix} -6 \\ 6 \end{pmatrix}.$$

(1 mark)

(ii) $\overrightarrow{OD}$

$\overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OD}$

$\begin{pmatrix} 9 \\ 0 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -6 \\ 6 \end{pmatrix}$

$\begin{pmatrix} 9 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$.

$\overrightarrow{OD}$ $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$.

(2 marks)

(iii) $\overrightarrow{BE}$

$\overrightarrow{OB} + \overrightarrow{BE} = \overrightarrow{OE}$

$\overrightarrow{BE} = \overrightarrow{OE} - \overrightarrow{OB}$

$= \frac{1}{3}\begin{pmatrix} 9 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

$\begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \end{pmatrix}$.

$BE = \begin{pmatrix} 0 \\ -6 \end{pmatrix}$.

(2 marks)

Total 12 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.