FORM TP 2024020

TEST CODE 01234020

JANUARY 2024

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of TWO sections: I and II.
2. Section I has SEVEN questions and Section II has THREE questions.
3. Answer ALL questions, writing your answers in the spaces provided in this booklet.
4. Numerical answers that are non-exact should be given correct to 3 significant figures or 1 decimal place for angles in degrees unless a different level of accuracy is specified in the question.
5. Do NOT write in the margins.
6. All working MUST be clearly shown.
7. **A list of formulae is provided on page 4 of this booklet.**
8. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
9. **If you use the extra page(s), you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**
10. **ALL diagrams in this booklet are NOT drawn to scale, unless otherwise stated.**

Required Examination Materials

Electronic calculator
Geometry set

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

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0 1 2 3 4 0 2 0 0 3

LIST OF FORMULAE

Volume of a prism — $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of a cylinder — $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.

Volume of a right pyramid — $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference — $C = 2\pi r$ where r is the radius of the circle.

Arc length — $S = \frac{\theta}{360} \times 2\pi r$ where θ is the angle subtended by the arc, measured in degrees.

Area of a circle — $A = \pi r^2$ where r is the radius of the circle.

Area of a sector — $A = \frac{\theta}{360} \times \pi r^2$ where θ is the angle of the sector, measured in degrees.

Area of a trapezium — $A = \frac{1}{2}(a + b)\,h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

Roots of quadratic equations — If $ax^2 + bx + c = 0$,

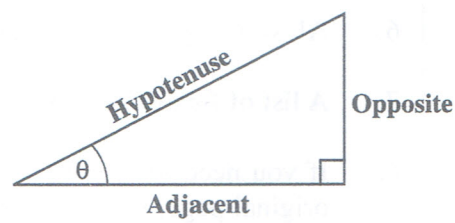
$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometric ratios

$$\sin\theta = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

$$\cos\theta = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$$

$$\tan\theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

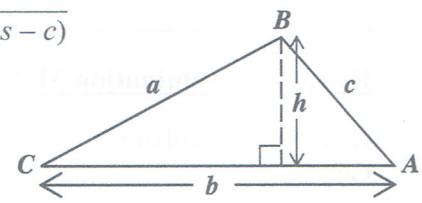


Area of a triangle — Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height.

$$\text{Area of } \Delta ABC = \frac{1}{2}ab\sin C$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$



Sine rule — $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule — $a^2 = b^2 + c^2 - 2bc\cos A$

GO ON TO THE NEXT PAGE

SECTION I

Answer ALL questions.

All working must be clearly shown.

1. (a) Express, as a single fraction in its **simplest** form,

$$1 - \left(\frac{1}{30} + \frac{4}{15}\right).$$

.....

(2 marks)

(b) A two-storey car park has a total of 1 020 parking spaces. At 06:30 hours one morning, $\frac{1}{30}$ of the 1 020 spaces are filled. During the next hour, no cars left the car park but another $\frac{4}{15}$ of the 1 020 spaces are filled. Determine the number of parking spaces that are NOT filled at 07:30 hours.

.....

(1 mark)

(c) Of the 1 020 parking spaces, 20% are on the top level. How many parking spaces are on the top level?

.....

(1 mark)

GO ON TO THE NEXT PAGE

(d) Some of the spaces are reserved for monthly paying customers. The ratio of reserved spaces to non-reserved spaces is 5:7.

Calculate the number of **non-reserved** parking spaces.

.....

(1 mark)

(e) The cost for parking at the car park is shown in the table below.

Length of Visit	**Cost ($)**
Under 30 minutes	Free
More than 30 minutes and up to 2 hours	$ 2.25
More than 2 hours and up to 4 hours	$ 5.50
More than 4 hours and up to 8 hours	$ 9.25
More than 8 hours and up to 24 hours	$15.00
One-week ticket	$40.00

(i) Mikayla leaves the car park at 18:30 hours and pays $9.25. Determine the **earliest** time she could have arrived at the car park.

.....

(1 mark)

(ii) Dhanraj bought a weekly parking ticket for $40. That week, he visited the car park five **different** times. The length of time he parked his car on each occasion is given below.

25 minutes $7\frac{1}{2}$ hours 11 hours $8\frac{3}{4}$ hours 8 hours

Show that Dhanraj saved $8.50 by buying the weekly ticket.

.....

(3 marks)

Total 9 marks

GO ON TO THE NEXT PAGE

2. (a) Simplify

$$\frac{x^2 + 7x}{x^2 - 49}.$$

.....

(2 marks)

(b) Find the value of

(i) r, when $x^2 \times x^6 = x^r$

.....

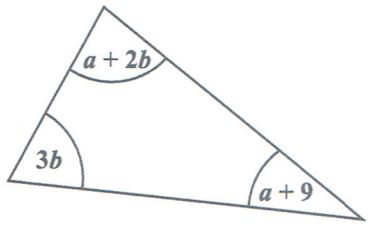
(1 mark)

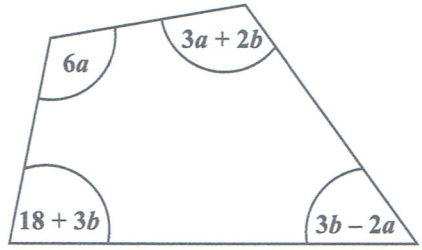
(ii) s, when $s^3 = 8$.

.....

(1 mark)

(c) The diagrams below show a triangle and a quadrilateral. All angles are in degrees and are written in terms of a and/or b.





(i) For the triangle, show that $2a + 5b = 171$.

.....

(1 mark)

(ii) For the quadrilateral, show that $7a + 8b = 342$.

.....

(1 mark)

(iii) Solve the pair of simultaneous equations in (i) and (ii) to find the values of a and b. **Show all working.**

.....

(3 marks)

Total 9 marks

GO ON TO THE NEXT PAGE

3. (a) In triangle ABC, $AC = 8$ cm and $BC = 5$ cm.

(i) Using a ruler and compasses only, construct triangle ABC. The line AB has been drawn for you.

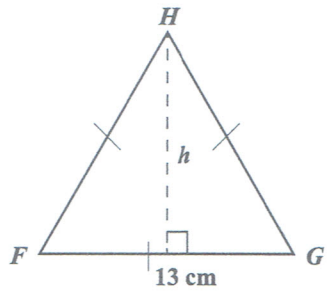
(2 marks)

(ii) Measure and state the value of Angle BAC.

.....

(1 mark)

(b) The diagram below shows an equilateral triangle, *FGH*, whose base is 13 cm and its height, *h*.

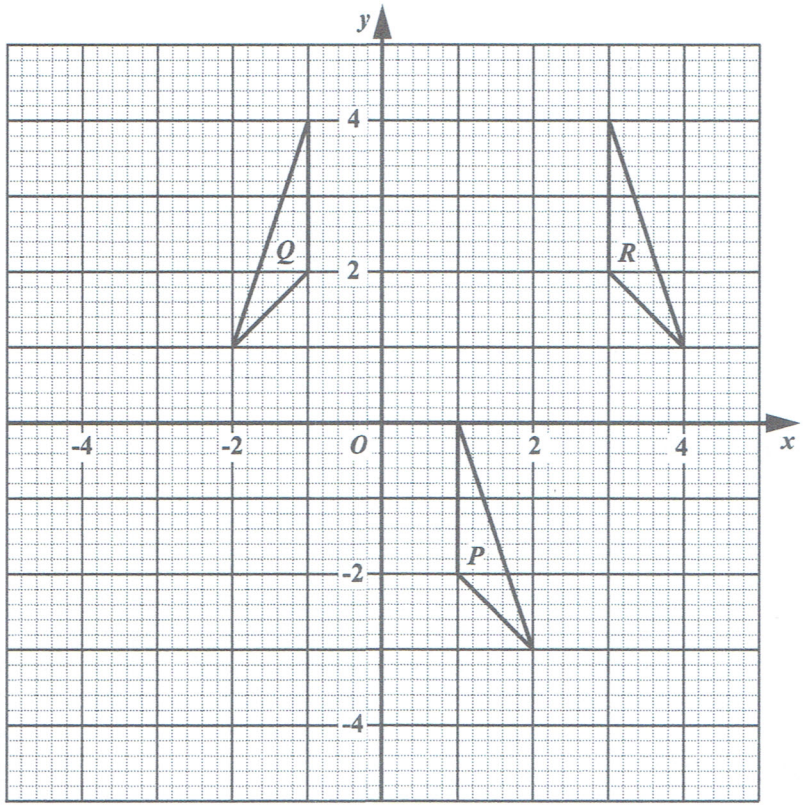


Calculate the value of *h*.

.....

(2 marks)

(c) The diagram below shows 3 triangles, P, Q and R. Triangles Q and R are images of Triangle P after it undergoes a double transformation.



Describe fully the single transformation that maps Triangle

(i) P onto Triangle R

.....

.....

.....

(2 marks)

(ii) R onto Triangle Q.

.....

.....

.....

(2 marks)

Total 9 marks

GO ON TO THE NEXT PAGE

4. Consider the following functions.

$$f(x) = 2 - 5x \text{ and } h(x) = 5^x.$$

(a) Calculate the value of

(i) $f(4)$

.....

(1 mark)

(ii) $h(0)$

.....

(1 mark)

(iii) $fh(-2)$.

.....

(2 marks)

(b) Find $f^{-1}(x)$.

.....

(2 marks)

(c) Given that $ff(x) = a + bx$, determine the values of a and b.

.....

(3 marks)

Total 9 marks

GO ON TO THE NEXT PAGE

NOTHING HAS BEEN OMITTED.

5. The mass, m, in kilograms, of 120 newborns at a hospital is recorded in the table below.

Mass (m kg)	**Frequency (f)**
$2.6 < m \leq 3.5$	7
$3.5 < m \leq 4.4$	18
$4.4 < m \leq 5.3$	30
$5.3 < m \leq 6.2$	29
$6.2 < m \leq 7.1$	28
$7.1 < m \leq 80$	8

(a) (i) State the modal class.

.....

(1 mark)

(ii) Complete the table below and calculate an estimate of the mean mass of the 120 newborns.

Mass (m kg)	**Midpoint (x)**	**Frequency (f)**	**Frequency × Midpoint (fx)**
$2.6 < m \leq 3.5$	3.05	7	21.35
$3.5 < m \leq 4.4$	3.95	18	71.1
$4.4 < m \leq 5.3$	4.85	30	145.5
$5.3 < m \leq 6.2$	_____	29	_____
$6.2 < m \leq 7.1$	6.65	28	186.2
$7.1 < m \leq 8.0$	_____	8	_____

.....

(3 marks)

GO ON TO THE NEXT PAGE

(iii) One newborn is chosen at random from the hospital. Find the probability that the newborn has a mass greater than 5.3 kg.

.....

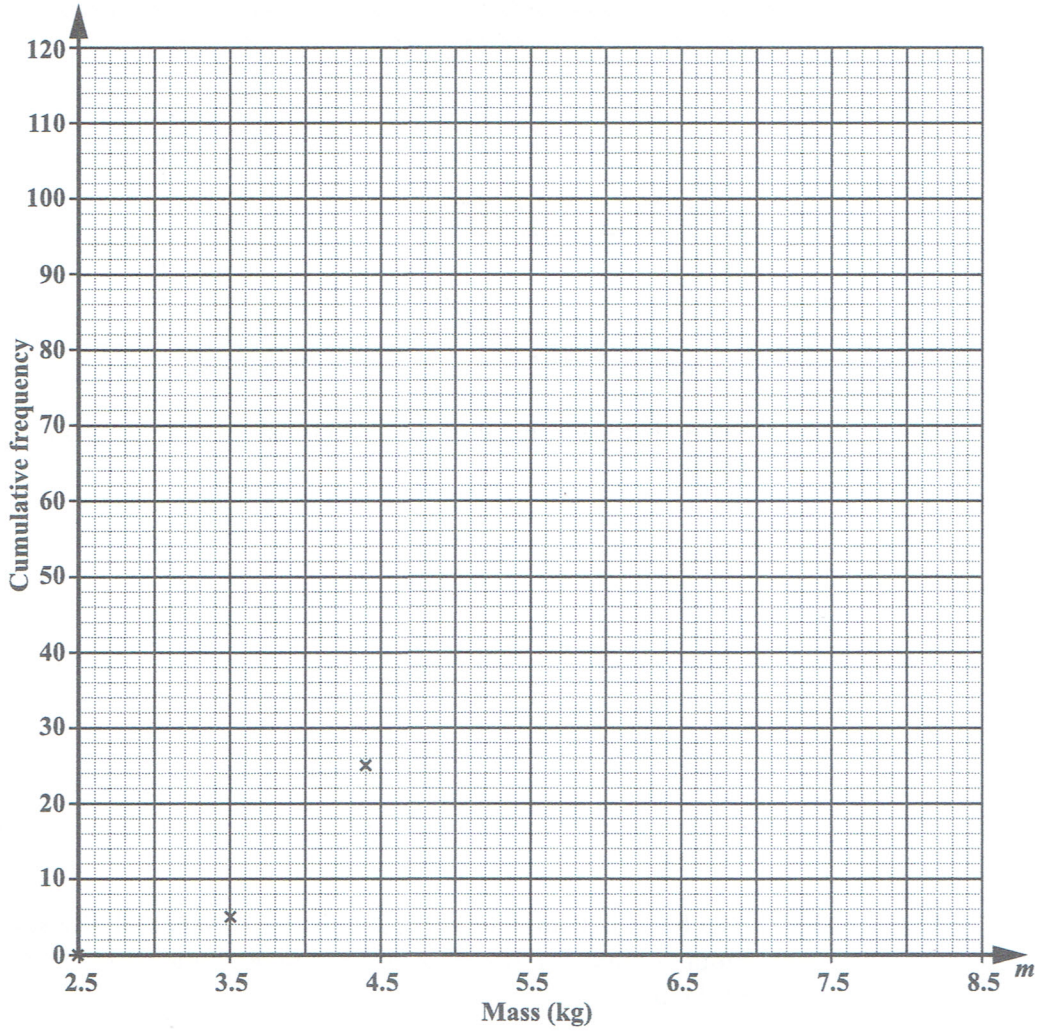
(1 mark)

(b) (i) Complete the cumulative frequency table shown below.

Mass (m kg)	**Cumulative Frequency**
$m \leq 3.5$	7
$m \leq 4.4$	25
$m \leq 5.3$	55
$m \leq 6.2$	_____
$m \leq 7.1$	_____
$m \leq 8.0$	_____

(1 mark)

(ii) On the grid below, draw a cumulative frequency curve to show the information in the table in (b) (i) **on page 16**.



(2 marks)

(iii) Use your diagram to find an estimate for the median mass of the newborns.

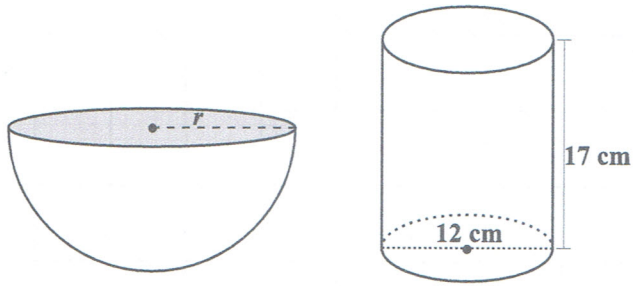
.....

(1 mark)

Total 9 marks

6. [In this question, use $\pi = \frac{22}{7}$.]

The diagrams below show a hemispherical bowl and a cylindrical tin. The diameter of the cylindrical tin is 12 cm, the height is 17 cm and the radius of the bowl is r.



(a) (i) Show that the volume of the cylindrical tin is 1 923 cm^3, correct to 4 significant figures.

.....

(2 marks)

(ii) The bowl is completely filled with soup. When all the soup is poured into the empty cylindrical tin, 90% of the volume of the tin is filled. Calculate the radius of the bowl.

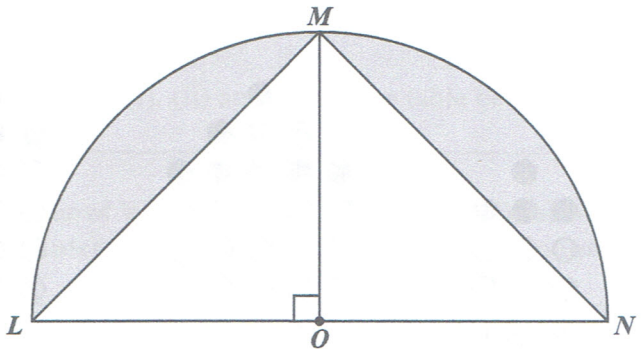
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

.....

(3 marks)

GO ON TO THE NEXT PAGE

(b) In the diagram below, points L, M and N are on the circumference of a semicircle, with centre O, and a radius of 18 cm.



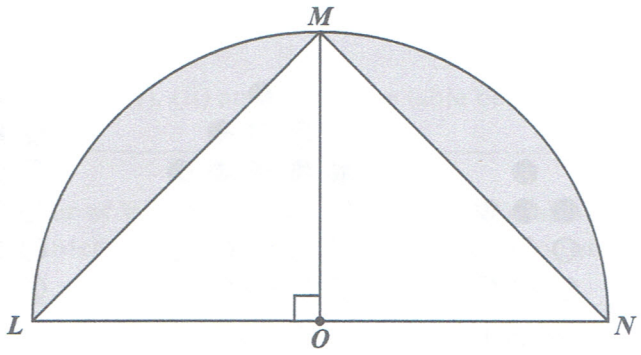
Calculate the TOTAL area of the shaded sections in the diagram.

.....

(4 marks)

Total 9 marks

(b) In the diagram below, points L, M and N are on the circumference of a semicircle, with centre O, and a radius of 18 cm.



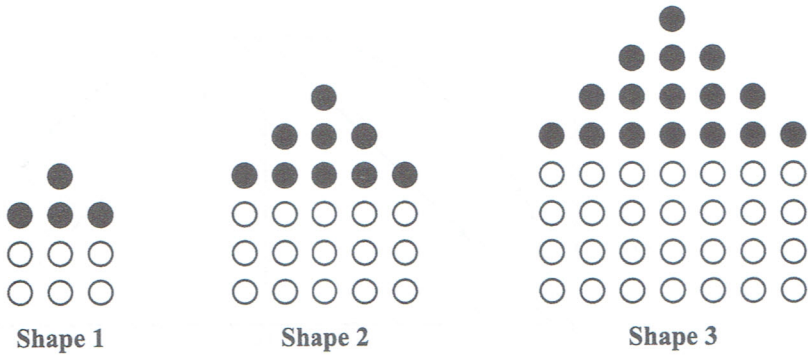
Calculate the TOTAL area of the shaded sections in the diagram.

.....

(4 marks)

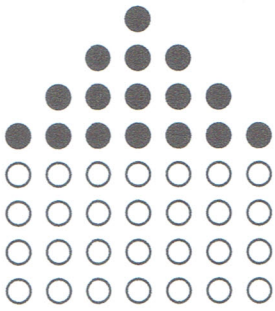
Total 9 marks

7. The diagram below shows the first 3 shapes in a sequence, which forms a pattern. Each shape is made using a set of small white counters and black counters.



Shape 1 **Shape 2** **Shape 3**

(a) Complete the diagram below to represent Shape 4.



.....

(2 marks)

(b) The number of white counters, W, the number of black counters, B, and the total number of counters, T, that form each shape follow a pattern. The values for W, B and T for the first 3 shapes are shown in the table below. Study the pattern of numbers in each row of the table and answer the questions that follow.

Complete the rows marked (i), (ii) and (iii) in the table below.

	Shape Number *(S)*	**Number of White Counters *(W)***	**Number of Black Counters *(B)***	**Total Number of Counters *(T)***	
	1	$(1 + 1)[2(1) + 1] = 6$	$(1 + 1)^2 = 4$	10	
	2	$(2 + 1)[2(2) + 1] = 15$	$(2 + 1)^2 = 9$	24	
	3	$(3 + 1)[2(3) + 1] = 28$	$(3 + 1)^2 = 16$	44	
(i)	4	**45**	_____	_____	**(2 marks)**
	⋮	⋮	⋮	⋮	
(ii)	_____	_____	**144**	**420**	**(2 marks)**
	⋮	⋮	⋮	⋮	
(iii)	n	(__ + __)[__ + __]	(__ + __)^2	$3n^2 + 5n + 2$	**(2 marks)**

(c) The expression for the total number of counters, $T = W + B$, in Shape S is given by $T = aS^2 + bS + 2$, where a and b are both positive integers.

By substituting suitable values for S, show that the **total** number of counters in Shape 1 and Shape 3, in terms of a and b, is represented by the equations

$$a + b = 8 \text{ and}$$
$$3a + b = 14 \text{ respectively.}$$

.....

(2 marks)

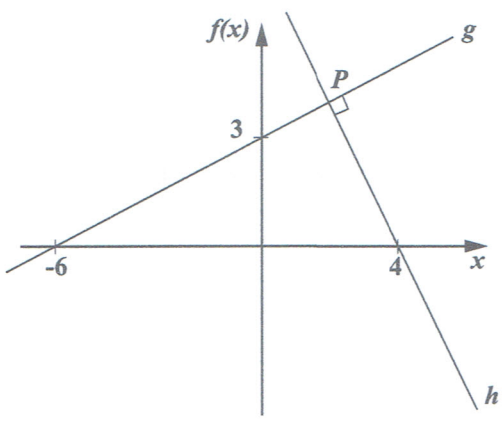
Total 10 marks

SECTION II

Answer ALL questions.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. (a) The diagram below shows the graphs of two functions on the same pair of axes. The lines g and h are perpendicular.



Determine the

(i) equation that represents the line g

.....

(2 marks)

(ii) equation that represents the line h

.....

(2 marks)

(iii) coordinates of the point P. **Show all working.**

.....

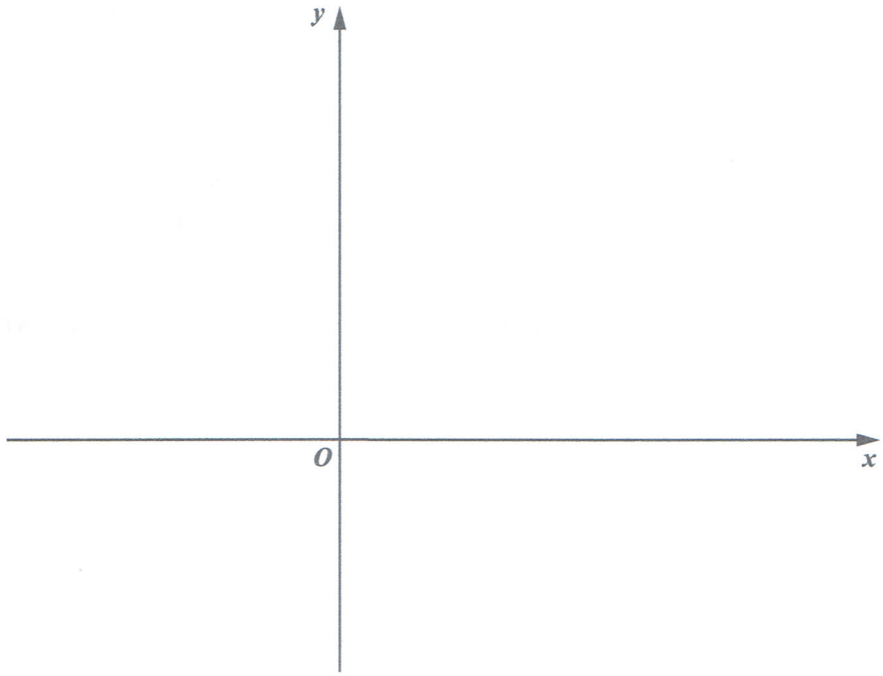
(2 marks)

(b) (i) Write $4x^2 - 24x + 31$ in the form $a(x + h)^2 + k$.

.....

(2 marks)

(ii) On the axes below, sketch the graph of $4x^2 - 24x + 31$, indicating the coordinates of the maximum/minimum point and the y-intercept.



(3 marks)

(iii) State the equation of the axis of symmetry.

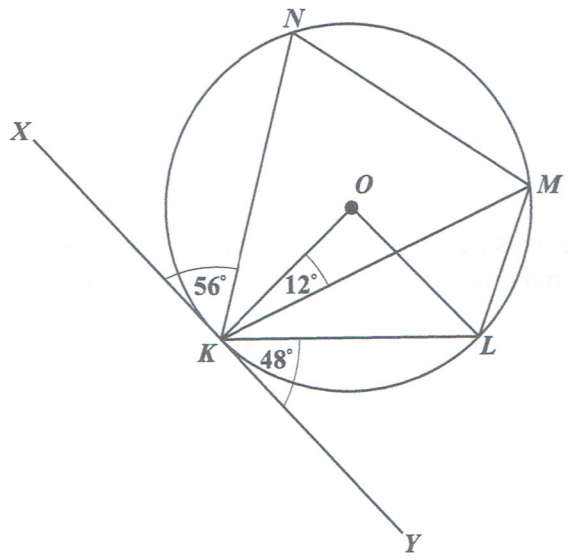
.....

(1 mark)

Total 12 marks

GEOMETRY AND TRIGONOMETRY

9. (a) *K, L, M* and *N* are points on the circumference of a circle with centre *O*. *XY* is a tangent to the circle at *K*. Angle *LKY* = 48°, *MKO* = 12° and Angle *XKN* = 56°.



(i) Find the value of Angle *KOL*, giving reasons for EACH step of your work.

Reason .....

.....

.....

(2 marks)

(ii) Find the value of EACH of the following angles.

a) Angle *LMN*

.....

(1 mark)

b) Angle *KLO*

.....

(1 mark)

c) Angle *MLK*

.....

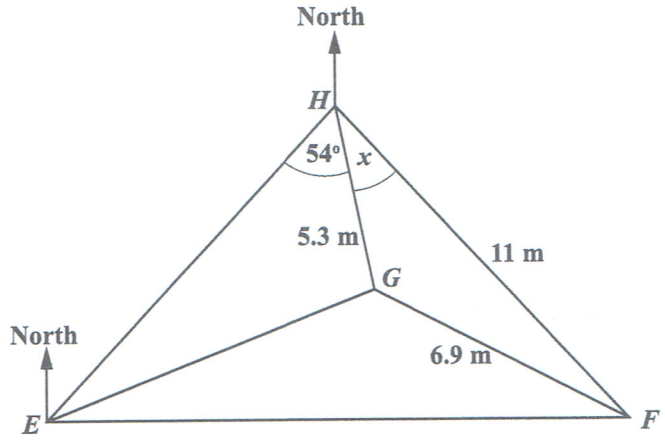
(1 mark)

d) Angle *KNM*

.....

(1 mark)

(b) *E, F, G* and *H* are 4 points on level ground. The diagram below gives information on the distances and angles between the points.



(i) Show that the value of x is 29.5°, correct to 1 decimal place.

.....

(2 marks)

(ii) A vertical tower, GT, is constructed at the point G and is pivoted to the ground at the points E, F and H using pieces of wire. The angle of elevation of the top of the tower, T, from the point F is 31°.

What length of wire was used to secure Point T to Point F?

.....

(2 marks)

(iii) The bearing of E from H is 228°. Find the bearing of

a) H from E

.....

(1 mark)

b) G from H.

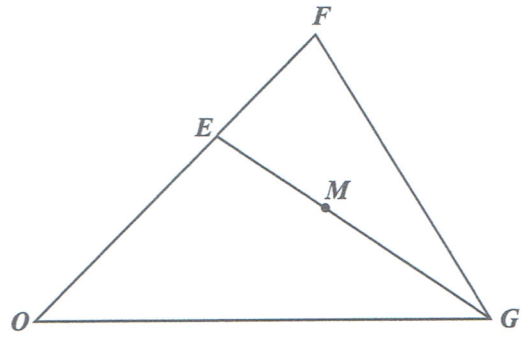
.....

(1 mark)

Total 12 marks

VECTORS AND MATRICES

10. (a) In the diagram below, O is the origin, $OE = 2EF$ and M is the midpoint of EG. $\overrightarrow{OG} = \mathbf{c}$ and $\overrightarrow{OF} = \mathbf{d}$.



Find in terms of **c** and **d**, in its simplest form,

(i) $\overrightarrow{FG}$

.....

(1 mark)

(ii) $\overrightarrow{EG}$

.....

(2 marks)

(iii) $\overrightarrow{OM}$.

.....

(2 marks)

(b) The matrices P, Q and R are given below, in terms of the scalar constants a, b and c, as

$$P = \begin{pmatrix} 3 & -9 \\ a & 7 \end{pmatrix}, \quad Q = \begin{pmatrix} -1 & b \\ -4 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} c & -3 \\ -4 & 8 \end{pmatrix}.$$

Given that $P + Q = R$, find the value of a, b and c.

.....

(3 marks)

(c) Solve the following pair of simultaneous equations using a matrix method.

$$5x - 2y = 44$$
$$2x + 3y = 10$$

.....

(4 marks)

Total 12 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.